

# Electric Current Multipole Moments in Classical Electrodynamics

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*The general theory for electric current multipoles appearing at the motion of magnetic dipoles and change in these values or orientation has been suggested. Static multipoles, including an anapole, have been studied in detail.*

## 1. Introduction

Electric current multipoles (ECM) occur at the motion of conductors carrying a current or at the variation of the current strength in conductors. While possessing a number of typical multipole properties, they have some interesting distinctions. Typically electric charge multipoles occur due to a particular spatial distribution of charge. ECM, as distinct from charge multipoles, are generated only by conductors carrying a current. Noncompensated charges are absent in the rest system of any conductor, and the charge density is equal to zero. There are two types of ECM. In laboratory system for the ECM occurring at the motion of conductors carrying a current the charge density is nonzero. Its appearance is due to the transformation of the charge and current densities as components of a four-vector. Just the charge density occurring at the motion of conductors determines electric multipole moments of the system. For this type of multipoles, electric current dipoles are known long ago [1], the classical theory of electric current quadrupoles is given in [2,3], their quantum theory is given in [4,5]. The existence of the second type of ECM has been suggested by Miller [6]. Multipoles of this type appear at the change of a current strength in a conductors or at the change of the spatial orientation of conductors carrying a current (or magnetic multipoles). The introduction of a magnetic current is a convenient approach for description

of these multipoles [6]. It is obvious that electric multipoles of the both types appear at the arbitrary motion of conductors carrying a current. This is a general case and will be analyzed in this work in terms of the classical electrodynamics.

The investigation of ECM is of a great practical importance. The electric current dipole moment occurring at the motion of a particle having a magnetic dipole moment determines the interaction of the particle's spin with an electrostatic field. The electric current quadrupole moment (ECQM) appearing at the orbital motion of nucleons can make a distinct contribution to the total quadrupole moments of nuclei [3–5]. As shown in [7], the moving anapole interacts with an inhomogeneous electric field. The investigation of this phenomenon is of a great interest because it concerns the non-contact interaction of the anapole with the static external field. As will be shown below, the interaction of an anapole with the inhomogeneous electric field is due to the possession of ECQM by a moving anapole.

## 2. The Field of Electric Current Multipoles

The electric current multipole moments of microscopic objects are most interesting for the investigation. For these objects, it is sufficient to investigate the motion of point magnetic dipoles rather than that of conductors of finite sizes, carrying a current. In case of microscopic objects, when the sizes of conductors carrying a current should be taken into account, the formulae obtained for magnetic dipoles can usually be applied with substitution a set of point magnetic dipoles for conductors carrying a current (the magnetic sheets model [8]).

We obtain the expression for the electric field generated by point magnetic dipoles. The strength for this field is defined by the formula:

$$\mathbf{E} = -\nabla\phi - \frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}. \quad (1)$$

Let  $\mathbf{r}$  be a radius-vector of a point where a point magnetic dipole is found, and let us define the strength of the field at a remote point having radius-vector  $\mathbf{R}$ , where  $|\mathbf{R}| \gg |\mathbf{r}|$ . Since we analyze the systems that can radiate electromagnetic waves, we emphasize that  $|\mathbf{R}| \ll \lambda$ , where  $\lambda$  is a characteristic wavelength of the radiation. In the reference system

where a dipole rests, a scalar potential  $\phi' = 0$ , and in the laboratory system

$$\phi' = (\mathbf{v} \cdot \mathbf{A})/c \equiv (\dot{\mathbf{r}} \cdot \mathbf{A})/c, \quad (2)$$

where  $\mathbf{v}$  is the velocity of the translational motion of a magnetic dipole. Since

$$\mathbf{A} = \frac{\boldsymbol{\mu} \times \mathbf{r}'}{r'^3}, \quad \mathbf{r}' = \mathbf{R} - \mathbf{r},$$

then by expansion of the quantity  $\mathbf{A}$  into a series we obtain:

$$\mathbf{A} = -[\boldsymbol{\mu} \times \nabla] \frac{1}{R} + (\mathbf{r} \cdot \nabla)[\boldsymbol{\mu} \times \nabla] \frac{1}{R} - \frac{1}{2}(\mathbf{r} \cdot \nabla)(\mathbf{r} \cdot \nabla)[\boldsymbol{\mu} \times \nabla] \frac{1}{R} + \dots + (-1)^{k+1} \frac{1}{k!} (\mathbf{r} \cdot \nabla)^k [\boldsymbol{\mu} \times \nabla] \frac{1}{R} + \dots, \quad (3)$$

where  $(\mathbf{r} \cdot \nabla)^k$  is a product  $k$  of cofactors  $(\mathbf{r} \cdot \nabla)$ , and  $[\boldsymbol{\mu} \times \nabla]f(R) \equiv [\boldsymbol{\mu} \times \nabla f(R)]$ . Taking the identity  $(\mathbf{v} \cdot [\boldsymbol{\mu} \times \nabla]) = ([\mathbf{v} \times \boldsymbol{\mu}] \cdot \nabla)$  into account, the electric field strength could be expressed as:

$$\mathbf{E} = \mathbf{E}^{(1)} + \mathbf{E}^{(2)} + \mathbf{E}^{(3)}, \quad (4)$$

$$\begin{aligned} \mathbf{E}^{(1)} &= -\nabla\phi, \quad \phi = \phi^{(1)} + \phi^{(2)} + \dots, \quad \phi^{(1)} = -(\mathbf{d} \cdot \nabla) \frac{1}{R}, \\ \phi^{(2)} &= -(\mathbf{r} \cdot \nabla)(\mathbf{d} \cdot \nabla) \frac{1}{R}, \dots, \quad \phi^{(k)} = (-1)^k \frac{1}{(k-1)!} (\mathbf{r} \cdot \nabla)^{k-1} (\mathbf{d} \cdot \nabla) \frac{1}{R}, \end{aligned} \quad (5)$$

$$\begin{aligned} \mathbf{E}^{(2)} &= \nabla \times \mathbf{G}, \quad \mathbf{G} = \mathbf{G}^{(0)} + \mathbf{G}^{(1)} + \mathbf{G}^{(2)} + \dots, \quad \mathbf{G}^{(0)} = -\frac{\dot{\boldsymbol{\mu}}}{cR}, \quad \mathbf{G}^{(1)} = \frac{1}{c} (\mathbf{r} \cdot \nabla) \frac{\dot{\boldsymbol{\mu}}}{R}, \\ \mathbf{G}^{(2)} &= -\frac{1}{2c} (\mathbf{r} \cdot \nabla)(\mathbf{r} \cdot \nabla) \frac{\dot{\boldsymbol{\mu}}}{R}, \dots, \quad \mathbf{G}^{(k)} = (-1)^{k+1} \frac{1}{ck!} (\mathbf{r} \cdot \nabla)^k \frac{\dot{\boldsymbol{\mu}}}{R}, \end{aligned} \quad (6)$$

$$\begin{aligned} \mathbf{E}^{(3)} &= \nabla \times \mathbf{N}, \quad \mathbf{N} = \mathbf{N}^{(1)} + \mathbf{N}^{(2)} + \dots, \quad \mathbf{N}^{(1)} = \frac{1}{c} (\mathbf{v} \cdot \nabla) \frac{\boldsymbol{\mu}}{R}, \\ \mathbf{N}^{(2)} &= -\frac{1}{c} (\mathbf{r} \cdot \nabla)(\mathbf{v} \cdot \nabla) \frac{\boldsymbol{\mu}}{R}, \dots, \quad \mathbf{N}^{(k)} = (-1)^{k+1} \frac{1}{c(k-1)!} (\mathbf{r} \cdot \nabla)^{k-1} (\mathbf{v} \cdot \nabla) \frac{\boldsymbol{\mu}}{R}, \end{aligned} \quad (7)$$

where  $\mathbf{d} = [\mathbf{v} \times \boldsymbol{\mu}]/c$ . Formulae (4)–(7) describe an electric field generated both at the translational motion of a magnetic dipole moment and at the change of its value or orientation. Indices for  $\phi, \mathbf{G}, \mathbf{N}$  define the rank of multipoles. The quantity  $-\frac{1}{c} \frac{\partial \mathbf{A}}{\partial t}$  in formula (1) corresponds to the sum  $\mathbf{E}^{(2)} + \mathbf{E}^{(3)}$ . The potential  $\phi^{(1)}$  coincides with a potential generated by a charge dipole moment  $\mathbf{d}$ , and formula (5) is the expansion in terms of multipoles. By separating quadrupole and contact interactions in the potential  $\phi^{(2)}$ , the latter can be expressed as:

$$\phi^{(2)} = \frac{1}{6} Q_{ij} \frac{\partial^2}{\partial X_i \partial X_j} \left( \frac{1}{R} \right) + \frac{1}{6} \tau \frac{\partial^2}{\partial X_i^2} \left( \frac{1}{R} \right), \quad (8)$$

$$Q_{ij} = 3x_i d_j + 3d_i x_j - 2\delta_{ij} x_k d_k, \quad \tau = 2x_k d_k. \quad (9)$$

As is known, the values  $\mathbf{d}$ ,  $Q_{ij}$  and  $\tau$  represent a dipole moment, a quadrupole moment tensor, and a mean square of a charge radius, respectively. For the system of charges

$$\mathbf{d} = \int \rho \mathbf{r} dV, \quad Q_{ij} = \int \rho (3x_i x_j - \delta_{ij} r^2) dV. \quad (10)$$

The fields  $\mathbf{E}^{(2)}$  and  $\mathbf{E}^{(3)}$  are vortex. For immobile magnetic dipoles ( $\mathbf{v} = 0$ )  $\mathbf{E}^{(1)} = \mathbf{E}^{(3)} = 0$ , and their electric field is the field of ECM described in [6]. The value  $\dot{\boldsymbol{\mu}}$  could be nonzero due to the change both of the magnetic moment value and direction of its orientation. The characteristic feature of this type of ECQM is the possibility of their description through the introduction of magnetic current of density  $\mathbf{j}^{(m)} = \sum_i \dot{\boldsymbol{\mu}}_i \delta(\mathbf{r} - \mathbf{r}'_i)$ . This allows to use mathematical tools of electrodynamics for magnetic charges [9].

Formula (6) gives the expansion of pseudovector potential  $\mathbf{G}$  in terms of degrees  $r/R$ . However, in the general case this formula does not allow to pass to such expansion in terms of multipoles typical for usual electric currents. In particular, the relation typical of the magnetic dipoles

$$(\mathbf{r} \cdot \nabla) \frac{\mathbf{j}}{R} = \frac{1}{2} [[\mathbf{r} \times \mathbf{j}] \times \nabla] \frac{1}{R},$$

is not fulfilled for dipole term  $\mathbf{G}^{(1)}$  in (6):

$$(\mathbf{r} \cdot \nabla) \frac{\dot{\boldsymbol{\mu}}}{R} = \dot{\boldsymbol{\mu}} (\mathbf{r} \cdot \nabla) \frac{1}{R} \neq \frac{1}{2} [[\mathbf{r} \times \dot{\boldsymbol{\mu}}] \times \nabla] \frac{1}{R}.$$

This is due to the fact that the value  $\dot{\boldsymbol{\mu}} = \int \mathbf{j}^{(m)} dV$  may not be given in the form of  $\dot{\boldsymbol{\mu}} = \int \rho^{(m)} \mathbf{v} dV$ , similar to the expression  $\mathbf{j} = \rho \mathbf{v}$ . However, the appropriate ECM appear in case of a particular configuration of magnetic currents, repeating the configuration of electric currents, generating different magnetic multipoles. The formulae for ECM in this case coincide with the formulae for magnetic multipoles at the substitution of  $\mathbf{E} \rightarrow -\mathbf{H}$ ,  $\mathbf{G} \rightarrow -\mathbf{A}$ ,  $\mathbf{j}^{(m)} \rightarrow \mathbf{j}$ . In particular, at the expansion in terms of multipoles, electric anapole (toroid dipole) moments appear in a natural way. Their existence has been suggested in [10,11]. The ECM field, with account of terms of the second order for  $r$ , is described by the expression:

$$\mathbf{E} = \nabla \times \mathbf{G}, \quad G_i = -e_{ijk} d'_j \frac{\partial}{\partial X_k} \left( \frac{1}{R} \right) + \left[ -\frac{1}{6} e_{ijn} Q_{nk} + \frac{1}{4\pi} (\delta_{ik} a'_j - \delta_{jk} a'_i) \right] \frac{\partial^2}{\partial X_j \partial X_k} \left( \frac{1}{R} \right), \quad (11)$$

where  $\mathbf{d}$ ,  $Q'_{nk}$  and  $\mathbf{a}'$  are the electric current dipole, quadrupole and anapole moments, respectively:

$$\mathbf{d} = -\frac{1}{2c}[\mathbf{r} \times \dot{\boldsymbol{\mu}}], \quad Q'_{nk} = \frac{1}{c}(e_{nlm}x_k + e_{klm}x_n)\dot{\mu}_lx_m, \quad \mathbf{a}' = \frac{\pi}{c}\dot{\boldsymbol{\mu}}r^2. \quad (12)$$

It is interesting to compare (11),(12) with the corresponding formulae for magnetic multipoles [12]:

$$\begin{aligned} \mathbf{H} = \nabla \times \mathbf{A}, \quad A_i = -e_{ijk}\mu_j \frac{\partial}{\partial X_k} \left( \frac{1}{R} \right) + \left[ -\frac{1}{6}e_{ijn}M_{nk} + \frac{1}{4\pi}(\delta_{ik}a_j - \delta_{jk}a_i) \right] \frac{\partial^2}{\partial X_j \partial X_k} \left( \frac{1}{R} \right), \\ \boldsymbol{\mu} = \frac{1}{2c} \int [\mathbf{r} \times \mathbf{j}] dV, \quad M_{nk} = \frac{1}{c} \int (e_{nlm}x_k + e_{klm}x_n)j_l x_m dV, \quad \mathbf{a} = -\frac{\pi}{c} \int \mathbf{j} r^2 dV, \end{aligned} \quad (13)$$

where  $\boldsymbol{\mu}$ ,  $M_{nk}$  and  $\mathbf{a}$  are the magnetic dipole, quadrupole and anapole moments.

The common property of ECM is the coincidence of their fields at great distances with the fields of corresponding charge multipoles.

Note the principle difference between the two types of ECM. The existence of ECM, appearing at the translational motion of conductors carrying a current, is related to the appearance of a nonzero density of charges:

$$\rho = \frac{\mathbf{j}_0 \cdot \mathbf{v}'}{c^2(1 - v'^2/c^2)^{1/2}} = \frac{\mathbf{j} \cdot \mathbf{v}'}{c^2}, \quad (14)$$

where  $\mathbf{v}'$  is the velocity of a conductor,  $\mathbf{j}_0$  is the current density in the rest system for a conductor carrying a current,  $\mathbf{j}$  is the current density in the laboratory system, being a sum of conduction and convection currents,  $\rho\mathbf{v}'$ . The appearing charges are real, since the relation  $\nabla \mathbf{E} = 4\pi\rho$  is satisfied. Note that the summed-up charge equals zero due to the law of charge conservation, at arbitrary motion of a conductor carrying a current in any reference system. The charge density equals zero in the entire space for ECM appearing at the change of value or direction of magnetic moments. These moments occur due to nonzero value  $\frac{1}{2\pi} \frac{\partial \mathbf{H}}{\partial t}$ , which might be called the magnetic displacement current by analogy with electric displacement current. The nonzero density of effective magnetic charges, defined by the formula similar to formula (14), occurs at the motion of the ECM of this type. However, magnetic charges and currents are just effective, and not real, since ECM are described by the typical Maxwell equations valid for  $\rho^{(m)} = 0$ ,  $\mathbf{j}^{(m)} = 0$ .

An essential distinction between the multipoles investigated in [1–5] and Miller multipoles [6] is related to the type of Lagrangian and Hamiltonian of the interaction. The field of ECM of the first type is caused by appearance of a nonzero charge density, and the scalar potential of this field is also nonzero. Therefore, external charges interact in the same way with the fields of charge and current electric moments, and in the both cases the Lagrangian of the interaction is equal  $\mathcal{L}_{int} = -e\phi$  ( $e$  is an external charge). The situation is different for the multipoles of second type (Miller multipoles) having unique electrodynamic properties. The scalar potential of their field is equal to zero, and the Lagrangian of the interaction of an external charge with the field of a multipole also equals zero for a charge at rest;  $\mathcal{L}_{int} = -\frac{e}{c}(\mathbf{v} \cdot \mathbf{A}) \rightarrow 0$ , when the velocity of the charge  $\mathbf{v} \rightarrow 0$ . However, despite this circumstance the charge and the Miller multipoles effectively interact. The force equal to  $\mathbf{F} = d\mathbf{p}/dt = e\mathbf{E}$  acts on the charge and affects the kinetic momentum  $\mathbf{p}$  of the charge, i.e. causes its motion. The expressions for  $\mathcal{L}_{int}$  and  $\mathbf{F}$  agree because the constancy of the generalized momentum  $\mathbf{P} = \partial\mathcal{L}/\partial\mathbf{t} = \mathbf{p} + e\mathbf{A}/c = const$  follows from the Lagrange equation:

$$\frac{d}{dt} \frac{\partial\mathcal{L}}{\partial\mathbf{v}} = \frac{d\mathbf{P}}{dt} = \frac{\partial\mathcal{L}}{\partial\mathbf{v}}$$

and the kinetic momentum  $\mathbf{p} = m\mathbf{v}/\sqrt{1 - v^2/c^2}$  varies due to the variation of the vector potential  $\mathbf{A}$ . So, though the Lagrangian and Hamiltonian of the interaction of the external resting charge with the Miller multipole are equal to zero, but their derivatives are nonzero and this charge begin motion in the Miller multipole field in the same way as it would move in the field of the corresponding charge multipole. The motion of the charge in the field of ECM of the both types outwardly does not differ from its motion in the charge multipole field, with the moment of the same value.

### 3. The Interaction of Electric Current Multipoles with the External Electric Field

The two types of ECM differ by the character of their interaction with the external electric field. The Lagrangian and Hamiltonian of the interaction of resting multipoles of the second type (Miller multipoles) with the electric field are equal to zero, because these

multipoles have no electric charge. However, here we also observe the unique properties of these multipoles. Under the influence of the electric field they begin to move, and their motion outwardly has no distinction from the motion of the corresponding charge multipoles [6].

The interaction of multipoles of the first type with the electric fields also has some interesting effects. The charge density, occurring at the motion of conductors carrying a current and described by formula (14), defines the type of the Lagrangian of the interaction:

$$\mathcal{L}_{int} = -d_i \frac{\partial \phi}{\partial X_i} - \frac{1}{6} Q_{ij} \frac{\partial^2 \phi}{\partial X_i \partial X_j} - \frac{1}{6} \tau \frac{\partial^2 \phi}{\partial X_i^2}, \quad (15)$$

where  $X_i$  is the coordinate of the center of the current system, and the values  $\mathbf{d}, Q_{ij}, \tau$  are given by formulae (10) where  $\rho$  is defined by formula (14) (with the substitution of  $\mathbf{v}$  for  $\mathbf{v}'$ ). Note that  $\partial^2 \phi / \partial X_i^2 \equiv \Delta \phi = -4\pi \rho_{ext}(0)$  where  $\rho_{ext}$  is the external charge density. In this case the distance to the current element  $\mathbf{r}' = \mathbf{R} + \mathbf{r}$  and  $\mathbf{v}' \equiv \dot{\mathbf{r}}' = \mathbf{V} + \mathbf{v}$ , where  $\mathbf{v} \equiv \dot{\mathbf{r}}, \mathbf{V} \equiv \dot{\mathbf{R}}$ . By taking  $\mathbf{v}$  beyond the symbol of the integral we find:

$$\begin{aligned} d_i &= \frac{V_k}{c^2} \int j_k x_i dV + \frac{1}{c^2} \int j_k v_k x_i dV = d_i^{(1)} + d_i^{(2)}, \\ Q_{ij} &= \frac{V_k}{c^2} \int j_k (3x_i x_j - \delta_{ij} r^2) dV + \int j_k v_k (3x_i x_j - \delta_{ij} r^2) dV = Q_{ij}^{(1)} + Q_{ij}^{(2)}, \\ \tau &= \frac{V_k}{c^2} \int j_k r^2 dV + \int j_k v_k r^2 dV = \tau^{(1)} + \tau^{(2)}. \end{aligned} \quad (16)$$

The first terms in (16) define ECM appearing only at the motion of magnetic multipoles with the velocity  $\mathbf{V}$ , the second terms describe ECM differing from zero in the rest system of the particle. Let us consider at first the first terms. We transform them, using the definitions of magnetic multipole moments (13). And for the anapole moment the following expression (see [13]) is equivalent to (13):

$$\mathbf{a} = \frac{2\pi}{c} \int (\mathbf{j} \cdot \mathbf{r}) \mathbf{r} dV. \quad (17)$$

It follows from (13), (17) that the formula for the anapole moment can be also given in the form:

$$\mathbf{a} = \frac{2\pi}{3c} \int [\mathbf{r} \times [\mathbf{r} \times \mathbf{j}]] dV. \quad (18)$$

Allowing for that the mean of the time derivative of the value varying in certain limits

is equal to zero, and  $j_k = \rho v_k \equiv \rho \dot{x}_k$  we obtain:

$$\begin{aligned}\left\langle \frac{d}{dt}(\rho x_k x_i x_j) \right\rangle &= \langle \dot{\rho} x_k x_i x_j \rangle + \langle j_k x_i x_j \rangle + \langle x_k j_i x_j \rangle + \langle x_k x_i j_j \rangle = 0, \\ \left\langle \frac{d}{dt}(\rho x_k x_i) \right\rangle &= \langle \dot{\rho} x_k x_i \rangle + \langle j_k x_i \rangle + \langle x_k j_i \rangle = 0.\end{aligned}$$

For stationary currents  $\rho = 0$ . By dropping angular brackets and allowing for the symmetry by the indexes  $i, j$ , we find:

$$j_k x_i x_j = \frac{2}{3}(j_k x_i x_j - x_k j_i x_j) = \frac{2}{3} e_{ikl} [\mathbf{r} \times \mathbf{j}]_l x_j = \frac{1}{3} e_{ikl} ([\mathbf{r} \times \mathbf{j}]_l x_j + [\mathbf{r} \times \mathbf{j}]_j x_l + [\mathbf{r} \times \mathbf{j}]_l x_j - [\mathbf{r} \times \mathbf{j}]_j x_l).$$

Hence,

$$\frac{1}{c} \left\langle \int j_k x_i x_j dV \right\rangle = \frac{1}{3} e_{ikl} M_{lj} + \frac{1}{2\pi} (\delta_{kj} a_i - \delta_{ij} a_k).$$

Similarly we can obtain:

$$\frac{1}{c} \left\langle \int j_k x_i dV \right\rangle = e_{ikl} \mu_l.$$

In view of the symmetry by the indices  $i, j$  it is easily found that

$$\begin{aligned}\mathbf{d}^{(1)} &= \frac{1}{c} [\mathbf{V} \times \boldsymbol{\mu}], \quad Q_{ij}^{(1)} = \frac{1}{2c} (e_{ikl} V_k M_{lj} + e_{jkl} V_k M_{li}) + \frac{1}{4\pi c} (3a_i V_j + 3a_j V_i - 2\delta_{ij} \mathbf{a} \cdot \mathbf{V}), \\ \tau^{(1)} &= -\frac{1}{\pi c} \mathbf{a} \cdot \mathbf{V}.\end{aligned}\tag{19}$$

Formulae (19) for ECM caused by the motion of magnetic dipoles and magnetic quadrupoles have been obtained in [1] and [2,3], respectively. Formulae (19) show as well that ECQM also appear as a result of the anapole motion. The interaction of the moving anapole with an electrostatic field is a very important nontrivial effect first found by Afanasiev [7]. In view of the fact that in Ref. [7] a more particular case of the anapole formed of point magnetic dipoles was considered, the formulae there obtained agree with (19). We remind that the field of the anapole has no effect on the motion of external charges because its strength is equal to zero ( $\mathbf{E} = \mathbf{H} = 0$ ). At the same time, the distinction of the vector potential from zero leads to the Aharonov-Bomh effect [13]. As is known, the anapole interacts with an external current at contact, and with an alternate electric field at distance [14].

When transferring to the quantum mechanical description, the anapole moment of a particle should be expressed through the pseudovector  $\mathbf{I}$  of the particle spin. According



to the formula  $\mathbf{a} = \frac{a}{I}\mathbf{I}$  [12] the anapole moment also becomes a pseudovector, and the interaction of the anapole with the electric field is  $P$ -odd.

The appearance of ECQM for the moving anapole and its interaction with the electrostatic field may lead to the observation and measurement of the electron anapole moment in the experiments with atoms. The  $P$ -odd interaction of the electron anapole moment with the electrostatic field of a nucleus is proportional to the nucleus charge, and the presence of this interaction can be observed in the precision experiments with heavy atoms, similar to the experiment where the anapole moment of  $^{133}\text{Cs}$  was first detected [15].

As seen from formulae (19), the moving anapole interacts at contact with external charges. This interaction is proportional to the value  $\tau$ . It is also  $P$ -odd and, in particular, contributes to the  $P$ -nonconservation in atoms.

The second terms in (16) describe nonzero ECM for the particles at rest. In the absence of the  $T$ -invariance violation the dipole moments (including the electric current moments) of atoms and nuclei are equal to zero. The ECQM of atoms and nuclei are nonzero, and for nuclei, as shown in [3–5], they are not small. Their contribution to the total quadrupole moments of some nuclei comprises the value of a few percent. The evaluations show that for the nucleus  $^{133}\text{Cs}$  the charge / current quadrupole moments are of the same order of magnitude [5]. We remind that the sum of the charge and current quadrupole moments is determined experimentally. The formulae for  $Q_{ij}^{(2)}$  and  $\tau^{(2)}$  are more convenient to be written for point magnetic dipoles. This expression agrees with the real physical pattern of this phenomenon for atoms and nuclei. For this purpose the replacement of  $\mathbf{r} \rightarrow \mathbf{r} + \mathbf{r}''$  where  $|\mathbf{r}''| \ll |\mathbf{r}|$  is sufficient. Then the vector  $\mathbf{r}$  describes the position of point magnetic dipoles,  $\mathbf{v} \equiv \dot{\mathbf{r}}$ , and the small vector  $\mathbf{r}''$  describes the position of current elements, constituting these dipoles, and  $\mathbf{j} = \rho \dot{\mathbf{r}}''$ . The integrals in (16) are equal to:

$$\frac{1}{c^2} \int j_k v_k x_i dV = \frac{1}{c} \sum e_{ikl} v_k \mu_l, \quad \frac{1}{c^2} \int j_k v_k x_i x_j dV = \frac{1}{c} \sum (e_{ikl} x_j + e_{jkl} x_i) v_k \mu_l. \quad (20)$$

The summing-up in (20) is done by different point magnetic dipoles. By averaging over time and in view of the mean values being equal to zero for the derivatives of the quantities

varying within finite limits with respect to time, we find:

$$\begin{aligned} \langle [\mathbf{v} \times \boldsymbol{\mu}] \rangle &= -\langle [\mathbf{r} \times \dot{\boldsymbol{\mu}}] \rangle, \quad \langle (e_{ikl}x_j + e_{jkl}x_i)v_k\mu_l \rangle = \\ &\left\langle -\frac{1}{2} ([\mathbf{r} \times \mathbf{v}]_i\mu_j + [\mathbf{r} \times \mathbf{v}]_j\mu_i) + \delta_{ij}[\mathbf{r} \times \mathbf{v}] \cdot \boldsymbol{\mu} - \frac{1}{2} (x_i[\mathbf{r} \times \dot{\boldsymbol{\mu}}]_j + x_j[\mathbf{r} \times \dot{\boldsymbol{\mu}}]_i) \right\rangle. \end{aligned} \quad (21)$$

Using formulae (16),(20),(21) and dropping angular brackets, we find:

$$\begin{aligned} \mathbf{d}^{(2)} &= -\frac{1}{c} \sum [\mathbf{r} \times \dot{\boldsymbol{\mu}}], \\ Q_{ij}^{(2)} &= \frac{1}{2c} \sum (3[\mathbf{r} \times \mathbf{v}]_i\mu_j + 3[\mathbf{r} \times \mathbf{v}]_j\mu_i - 2\delta_{ij}[\mathbf{r} \times \mathbf{v}] \cdot \boldsymbol{\mu} + 3x_i[\mathbf{r} \times \dot{\boldsymbol{\mu}}]_j + 3x_j[\mathbf{r} \times \dot{\boldsymbol{\mu}}]_i), \\ \tau^{(2)} &= \frac{2}{c} \sum ([\mathbf{r} \times \mathbf{v}] \cdot \boldsymbol{\mu}). \end{aligned} \quad (22)$$

By introducing the orbital moment  $\mathbf{l} = [\mathbf{r} \times \mathbf{p}] = m_{rel}[\mathbf{r} \times \mathbf{v}]$  ( $m_{rel} = m/\sqrt{1-v^2/c^2}$  is the relativistic mass) we transform (22) as follows:

$$\begin{aligned} Q_{ij}^{(2)} &= -\frac{1}{2m_{rel}c} \sum \{3l_i\mu_j + 3l_j\mu_i - 2\delta_{ij}\mathbf{l} \cdot \boldsymbol{\mu} + 3m_{rel}(x_i[\mathbf{r} \times \dot{\boldsymbol{\mu}}]_j + x_j[\mathbf{r} \times \dot{\boldsymbol{\mu}}]_i)\}, \\ \tau^{(2)} &= \frac{2}{m_{rel}c} \sum (\mathbf{l} \cdot \boldsymbol{\mu}). \end{aligned} \quad (23)$$

The value  $\dot{\boldsymbol{\mu}}$  in formulae (22),(23) for atoms (nuclei) is determined by the electron (nucleon) spin precession. For the states with the orbital moment  $l \neq 0$ , the terms containing  $\dot{\boldsymbol{\mu}}$  can be neglected [2,3]. In this case the formulae for ECQM coincide with those given in these works. For  $l = 0$  ( $s$ -states) the terms containing  $\dot{\boldsymbol{\mu}}$  should be taken into account in the calculations.

The term proportional to  $\dot{\boldsymbol{\mu}}$  in formula (23) for ECQM agrees to the accuracy of the factor 3/2 with expression (12) for the quadrupole moment created by the magnetic current  $\mathbf{j}^{(m)} = \sum_i \dot{\boldsymbol{\mu}}_i \delta(\mathbf{r} - \mathbf{r}_i)$ . The presence of this additional factor in (23) is not surprising, because when averaging over time as it has been done in the derivation of formulae (22),(23), we determine the mean field at the fixed point of space. And in this case  $\left\langle \frac{\partial \mathbf{A}}{\partial t} \right\rangle = \left\langle \frac{d\mathbf{A}}{dt} \right\rangle = 0$ , and there is no contribution of the Miller multipoles to the mean field of the current system. The Miller multipoles do not as well contribute to the charge density the distribution of which determines static quadrupole moments. However, we can say that the static ECQM of the magnetic dipole system is the sum of the quadrupole moments caused by the orbital motion of magnetic dipoles and by the variation of the magnetic dipole moment values and their orientation.

The value  $\tau$  defined by formula (23) describes the contribution of the electrostatic current contact interaction (ECCI) to the value of the total root-mean-square radius of the system. The ECCI does not depend on  $\boldsymbol{\mu}$  in the system of magnetic dipoles. The classical expression for the ECCI to the accuracy of the constant factor corresponds to the quantum mechanical expression obtained in [4].

#### 4. Conclusion

The general formulae for the ECM appearing in the system of conductors carrying a current or magnetic dipoles have been obtained in terms of the classical electrodynamics. There are electric current multipole moments of the two types: 1) the moments appearing at the translational motion of conductors (magnetic multipoles); 2) the moments appearing at the current strength variation in conductors or at the change of the magnetic dipole orientation. The multipoles of the first type are created by electric charges appearing at the motion of conductors carrying a current (magnetic multipoles). ECM of this type appear both at the motion of particles having magnetic multipole moments and in the rest system of compound particles. The most important consequence of their existence is a sufficiently great ECQM values for some nuclei in comparison with their total quadrupole moments and the appearance of ECQM for the moving anapole leading to the  $P$ -nonconservation at the interaction of the particles having an anapole moment with an external electrostatic field. For the multipoles of the first type electric charges are absent, and effective magnetic currents can be introduced at the description of their field. For ECQM of the both types the character of their motion in the electric field and of the external charges in the ECM field is identical and remains the same as for the electric charge multipoles. At averaging the electric field over time only the multipoles of the first type remain. The relationships obtained for them can be transformed by introducing the orbital moment. In particular, as a result, the dependence of ECQM both on the orbital motion of magnetic dipoles and the variation of their moment values or change of their spatial orientation arises. The orbital motion of magnetic dipoles causes the appearance of ECCI influencing the value of a charge root-mean-square radius of the system.

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